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ON THE HYDROMAGNETIC STABILITY OF STRATIFIED FLUID IN THE PRESENCE OF SUSPENDED PARTICLES

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ABSTRACT

The paper critically examines within the framework of linear stability analysis, the hydromagnetic stability of stratified fluid in the presence of suspended particles. In the present paper, by a number of theorems providing conditions for stability or instability. This paper also shows the dual character of magnetic field. A particular case of uniform magnetic field is also discussed.

KEYWORDS: Hydromagnetic stability, Stratified fluid, Suspended particles.

INTRODUCTION

A comprehensive account of the stability of stratified fluids under varying assumptions regarding density, viscosity and magnetic field has been given by Chandrashekhar **[1]**. Chandra **[2]** observed a contradiction between the theory for onset of convection in fluids heated from below in his experiment. He performed the experiment in an air layer and found that the instability depended on the depth of the layer. A Benard **[3]** type cellular convection with fluid descending at the cell centre was observed when the predicted gradients were imposed for layers deeper than 10 mm. A convection which was different in character from that in deeper layers occurs at much lower gradients than predicted, if the layer depth was less than 7 mm. This is called the columnar instability. He added an aerosol to mark the flow pattern. Motivated by interest in fluid-particle mixtures and columnar instability, Scanlon and Segel **[4]** studied the effect of suspended particles on the onset of Benard convection and showed that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. Sharma et al. **[5]** considered the effect of suspended particles and found them to destabilize the layer whereas the effect of magnetic field was stabilizing. Generally, the magnetic field has a stabilizing effect on the instability but a few exceptions are there. For example, Kent **[6]** studied the effect of a horizontal magnetic field which varies in the vertical direction, on the stability of parallel flows and showed that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable.

The problem of the hydromagnetic stability of conducting fluid of variable density in the presence of suspended particles plays an important role in the stability of stellar atmosphere. The effect of suspended particles on the stability of stratified fluids might be of industrial and chemical engineering importance. Further motivation for this study is the fact that knowledge concerning fluid-particle mixtures is not commensurate with their industrial and scientific importance. Sharma and Singh **[7]** have investigated the stability of a fluid-particle mixture with variable density and viscosity in the presence of a variable magnetic field. The heterogeneous fluid of zero resistivity is under the action of gravity $\mathbf{g}(0, 0, -g)$ and is acted upon by a variable horizontal magnetic field.

This paper critically re-examines the work of Sharma and Singh and points out and corrects the serious error committed by them in their mathematical analysis of both the viscous and non-viscous theory. Contrary to their claim, it has been shown that the criteria determining stability or instability definitely depend upon the viscosity and dust particles.

In order to explain the error committed by Sharma and Singh, some explanation of the origin of extra force term KN $(\mathbf{U} - \mathbf{V})$, as has been given here, is necessary.

When the dynamics of one or several particles immersed in a fluid is considered, it becomes immaterial whether the fluid is moving past the solid or the solid is moving through the fluid. When a solid is immersed in a system of real fluid, because of viscous shear and friction, a force is required in order to maintain the relative flow between the solid and the fluid. By virtue of the principle of action and reaction, the force exerted by the solid on the fluid must be equal and opposite to the force exerted by the fluid on the solid. This force is called the drag force and is a function of the relative velocity between the solid and fluid, whereas the other two forces acting on any particle immersed in a fluid are gravity and buoyancy (opposite to each other) and are independent of the relative velocity of the solid with respect to the fluid.

Sedimentation is yet another important phenomenon occurring in fluid-particle mixture. In a gravitational field of force, a particle suspended in a less dense liquid medium tends to migrate through the fluid in a downward direction (This phenomenon is known as sedimentation). The knowledge of sedimentation may help in the determination of the size and the mass of the solute particles. In some cases, due to backward diffusion, equilibrium might occur. Marble **[8]** has critically examined the fundamental equations for dusty gas flows. It is well known that the viscosity of dusty gas should be increased by a factor proportional to the concentration by volume of the dust particles (Einstein formula for the viscosity of a suspension), a dust particle in air, or in any other gas, has a much larger inertia than the equivalent volume of air, the relative motion of the dust particles and air well dissipate energy because of the drag between dust and air, and the critical Reynold's number for transition from laminar to turbulent flow is affected by the dust particles. Saffman **[9]** has provided mechanism of particle gas mixture in the context of the stability of dusty flows. He investigated the effect of dust in terms of two parameters, the concentration of dust and a relaxation time. He showed that Reynolds number is reduced by a factor (1 + f) for fine dust and is increased for coarse dust. It is also known that an increase in the size of coarse dust particles reduces the stabilizing effect. The study of the stability of gas-particle mixture has, among others, been undertaken by Kochcr **[10]**, who investigated the stability of inviscid parallel shear flow of gas and dust.

The above discussion explains how the flow mechanism of gas-dust mixture differs from that of clean gas. The presence of dust particles (fine or coarse) also affects the flow instability irrespective of whether the gas is assumed to be viscous or non-viscous. The fact is that the non-viscous fluid is an idealization of the situations of fluids with small viscosity, as such the extra drag force KN (**U** – **V**) cannot be ignored even for a non-viscous gas in the presence of dust particles. This drag force, can, however, be small, depending upon the Stoke's resistance coefficient K (= 6 $\pi\mu\eta$, η being the radius of the particles assumed to be spherical and N being the number density of dust particles). Even for slightly viscous fluids (□ small), the value of K and hence the drag force may not be small (note that the radius □ for coarse dust may not be small). Moreover, the relaxation time for coarse dust is greater than the characteristic time of the disturbances. In this case, the dual perturbation vanishes and the coarse dust does not move with the gas when the flow is perturbed but carries on with the velocity of basic flow, so that the net effect of the dust added to the gas flow is equivalent to an extra frictional force proportional to the velocity and in no case this extra force can be ignored. In the process of setting final stability governing equations for non-viscous case, Sharma and Singh could not resist the temptation of setting K = 0 so that n´ was replaced by n and was taken outside the derivative sign.

In doing so, they committed a serious error by neglecting the drag force KN(**U** – **V**) in the equations of motion for gas and the dust particles. In fact, by setting K and thus by setting the drag force equal to zero, the dust gas interaction is completely ignored as is apparent from equations (1) and (6) given below and the problem investigated by Sharma and Singh essentially remains the one for clean gas.

EQUATIONS OF MOTION

Consider a static state in which an incompressible fluid-particle layer is arranged in horizontal strata and the pressure p, density □ and viscosity □ are the functions of the vertical coordinate z only. The character of the equilibrium of this static state is determined by supposing that the system is slightly disturbed and then following the further evolutions of perturbations.

Let □, □, p, □$_e$ and **V**(u, v, w) denote respectively the density, viscosity, pressure, magnetic permeability and the velocity of the pure fluid; **U**(x, t) and N(x, t) respectively denote the velocity and the number density of the particles.

$K = 6\pi\mu\eta$, where η is the particle radius, is a constant and $\bar{\lambda} = (0,0,1)$. Then the equations of motion and continuity for the fluid are

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla)\mathbf{V}\right] = -\nabla p - \rho g\bar{\lambda} + \nabla.(\mu\nabla\mathbf{V}) + (\nabla\mu\nabla)\mathbf{V} + KN(\mathbf{U}-\mathbf{V}) + \frac{\mu_e}{4\pi}(\nabla\times\mathbf{H})\times\mathbf{H} \quad (1)$$

$$\nabla.\mathbf{V} = 0 \quad (2)$$

$$\frac{\partial \mathbf{H}}{\partial t} = (\mathbf{H}.\nabla)\mathbf{V} - (\mathbf{V}.\nabla)\mathbf{H} \quad (3)$$

and $$\nabla.\mathbf{H} = 0 \quad (4)$$

Since the density of a fluid particle moving with the fluid remains unchanged, we have

$$\frac{\partial \rho}{\partial t} + (\mathbf{V}\nabla)\rho = 0 \quad (5)$$

In the equation of motion (5), the presence of particles adds an extra force term, proportional to the velocity difference between particles and fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude, but opposite in sign, in the equation of motion of the particles, the distances between the particles are quite large as compared to their diameter,
Inter-particle reactions are also not considered as we assume that the distances between the particles are large as compared to their diameter.

The equations of motion and continuity, under the above assumptions, are

$$mN\left[\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U}.\nabla)\mathbf{U}\right] = -mNg\bar{\lambda} - KN(\mathbf{V}-\mathbf{U}) \quad (6)$$

and $$\frac{\partial \mathbf{N}}{\partial t} + \nabla.(N\,\mathbf{U}) = 0, \quad (7)$$

where mN is the mass of particles per unit volume.

BASIC STATE AND THE PERTURBATION EQUATIONS

The initial stationary state, whose stability we wish to examine, is that of an incompressible fluid arranged in a horizontal strata in a heterogeneous medium. The system is acted upon by a magnetic field $\mathbf{H}[H_0(z), 0, 0]$ and the gravity field $\mathbf{g}(0, 0, -g)$.

Character of equilibrium of the initial static state is determined by supposing that the system is slightly disturbed and then by following its further evolutions.

Let $\delta\rho, \delta p, \mathbf{v}(u, v, w), \mathbf{u}(l, r, s)$ and $\mathbf{h}(h_x, h_y, h_z)$ denote respectively the perturbations in density □, pressure p, fluid velocity (0, 0, 0) and magnetic field $\mathbf{H}(H_0(z), 0, 0)$.
Then the linearized hydromagnetic perturbation equation are

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p + \mu\nabla^2 u + \frac{d\mu}{dz}\left(\frac{du}{dz} + \frac{\partial w}{\partial x}\right) + \frac{\mu_e}{4\pi}h_z DH_0 + KN(\mathbf{u}-\mathbf{l}) \quad (8)$$

$$\rho\frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p + \mu\nabla^2 v + \frac{d\mu}{dz}\left(\frac{dv}{dz} + \frac{\partial w}{\partial y}\right) + \frac{\mu_e}{4\pi}\left(\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}\right)H + KN(\mathbf{v}-\mathbf{r}) \quad (9)$$

$$\rho\frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p + \mu\nabla^2 w + \frac{d\mu}{dz}\left(\frac{dw}{dz} + \frac{\partial w}{\partial z}\right) - g\delta\rho + + \frac{\mu_e}{4\pi}\left(\frac{\partial h_z}{\partial x} - \frac{\partial h_x}{\partial z} - h_x DH_0\right) + KN(\mathbf{w}-\mathbf{s}) \quad (10)$$

$$\nabla.\mathbf{v} = 0, \quad (11)$$

$$\nabla.\mathbf{h} = 0, \tag{12}$$

$$\frac{\partial \mathbf{h}}{\partial t} = (\mathbf{H}.\nabla)\mathbf{v} - (\mathbf{v}.\nabla)\mathbf{H}, \tag{13}$$

$$mN\frac{\partial \mathbf{u}}{\partial t} = KN(\mathbf{v} - \mathbf{u}) \tag{14}$$

and $$\frac{\partial}{\partial t}(\delta\rho) = -w\frac{d\rho}{dz}. \tag{15}$$

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y and t is given by

$$\exp[i(k_x x + k_y y) + nt] \tag{16}$$

where n is general, is a complex constant; k_x and k_y are wave numbers along x and y directions respectively and $k^2 = k_x^2 + k_y^2$.

With this dependence of physical variables on x, y and t, we have after using the form of perturbations in (16). Now apply the usual procedure we may get final stability governing equation for viscous fluid as

$$D\left[\rho n'.Dw - \mu(D^2 - k^2)Dw - (D\mu)(D^2 + k^2)w + \frac{i\mu_e k_x}{4\pi}.DH_0 z - \frac{k_x k_y \mu_e H_0^2 \tau}{4\pi n} - \frac{\mu_e H_0 k_y^2}{4\pi n} w(DH_0)\right]$$

$$= k^2\left[\rho n' w - \mu(D^2 - k^2)w - 2(D\mu)(Dw) - \frac{g}{n}(D\rho)w - \frac{\mu_e}{4\pi}H_0\left(ik_x h_z - Dh_x - h_x\frac{DH_0}{H_0}\right)\right] \tag{17}$$

DISCUSSION

The final stability governing equation when the fluid is non-viscous is given by

$$D[n'\rho Dw - n'k^2\rho w] + \frac{gk^2}{n}(D\rho)w = -\frac{\mu_e k_x^2}{4\pi n}[H_0^2(D^2 - k^2)w + D(H_0^2)Dw]. \tag{18}$$

The boundary conditions are given by w = 0 at z = 0 and d.

Substituting for $n'\left(= n\left\{1 + \frac{mN}{\rho\left[1 + \frac{mN}{K}\right]}\right\}\right)$ in equation (18), we get

$$n^3 M[\rho(D^2 - k^2)w + (D\rho)Dw] + n^2[(\rho + mN)(D^2 - k^2)w + D(\rho + mN)Dw]$$
$$+ n\left[gk^2(D\rho)Mw + \frac{M\mu_e k_x^2}{4\pi}\{H^2(D^2 - k^2)w + DH^2.Dw\}\right]$$
$$+\left[gk^2(D\rho)w + \frac{\mu_e k_x^2}{4\pi}\{H^2(D^2 - k^2)w + DH^2 Dw\}\right] = 0 \tag{19}$$

where $M = \frac{m}{K}$ and $K = 6\pi\mu\eta$.

Following Sharma and Singh **[7]**, we assume that the magnetic field **H** and the stratifications in density □ and the number N of dust particles are of the form

$$H^2 = H_0^2 e^{\beta z}, \rho = \rho_0 e^{\beta z} \quad \text{and} \quad N = N_0 e^{\beta z}. \tag{20}$$

where H_0, ρ_0, N_0 and β are constants.

Now, substituting for H, □ and N from equation (20) into equation (19), introducing the non-dimensional quantities □, D* and a defined by

$$\eta = \frac{\sigma\nu}{d^2}, D^* = dD, a = kd$$

and dropping the asterisk '*' for convenience, the final stability governing equation (19) becomes

$$M_1\sigma^3[\beta Dw + (D^2 - a^2)w] + \sigma^2 f'[\beta Dw + (D^2 - a^2)w]$$
$$+[M_1\sigma + 1][Ra^2\beta w + Qa_x^2\{\beta Dw + (D^2 - a^2)w\}] = 0. \tag{21}$$

where $R = \frac{gd^3}{\nu^2}, M_1 = \frac{M\nu}{d^2}, Q = \frac{\mu_e H_1^2 d^2}{4\pi\rho_0\nu^2}, f' = 1 + f$ and $f = \frac{mN_0}{\rho_0}$.

Alternatively, equation (21) can be written as

$$AD^2w + BDw + Cw = 0, \tag{22}$$

where

$$A = [M_1\sigma^3 + f'\sigma^2] + Qa_x^2[1 + M_1\sigma], B = [M_1\sigma^3 + f'\sigma^2]\beta + Qa_x^2\beta[1 + M_1\sigma] = \beta.A$$
and $C = (1 + M_1\sigma)(Ra^2\beta - Qa_x^2a^2) - M_1\sigma^3a^3 - f'a^2\sigma^2.$

The boundary conditions are : w = 0 at z = 0 and 1.
Solution of equation (22) is given by

$$w = A_0 \exp[□_1 z] + A_1 \exp[□_2 z]. \tag{23}$$

where A_0 and A_1 are two arbitrary constants and $□_1$ and $□_2$ are the roots of the equation

$$A□^2 + B□ + C = 0 \tag{24}$$

The vanishing of w at z = 0 leads to

$$w = A_0 [\exp(□_1 z) - \exp(□_2 z)]. \tag{25}$$

where as the vanishing of w at z = 1 leads to

$$\exp(□_1 - □_2) = 0 \tag{26}$$

which implies that

□□□□□□□□□□□□□□□□□□□□$□_1 - □_2 = 2iS$□ (27)

where S is a positive integer.
Now from equations (24) and (25), we have

$$B^2 - 4AC + LA^2 = 0$$

Or $[B^2 + L]A = 4C$ where $L = 4S^2□^2$, (28)

Substituting for A and C in equation (22), we get

$$A'□^3 + B'□^2 + C'□ + D' = 0, \tag{29}$$

where

$$A' = (\beta^2 + L + 4a^2)M_1,$$
$$B' = (\beta^2 + L + 4a^2)f',$$
$$C' = [(\beta^2 + L)Qa_x^2 + 4a^2(Qa_x^2 - R\beta)]M_1$$
and $D' = (\beta^2 + L)Qa_x^2 + 4a^2(Qa_x^2 - R\beta).$

RESULTS

Theorem 1 : The system is stable if □ < 0.

Proof : If we assume that □ < 0, then equation (29) does not allow any positive root and therefore the system is stable for disturbances of all wave numbers.

Cor. : For □ > 0, the system is stable under the condition $Qa_x^2 > R\beta$.

Theorem 2 : Unstable modes exist under the conditions

$$\square < 0 \quad \text{and} \; Q < \frac{4R\beta}{\beta^2 + L + 4a^2}$$

and are non-oscillatory.

Proof : Under the conditions of the theorem, D´ is negative and therefore if □₁, □₂ and □₃ are the roots of equation (29), then

$$\sigma_1 + \sigma_2 + \sigma_3 = -\frac{B'}{A'} < 0 \text{ since } A' > 0 \text{ and } B' > 0,$$

$$\text{and} \quad \sigma_1 . \sigma_2 . \sigma_3 = -\frac{D'}{A'} > 0 \text{ since } D' < 0.$$

Since the sum of the roots is negative and the product is positive, therefore one root of the equation is positive which implies the instability of the system under the conditions of the theorem.

Further, taking the imaginary part of equation (29) after its division by □, we get

$$\sigma_i \left[2\sigma_r A' + B' - \frac{D'}{|\sigma|^2} \right] = 0 \qquad (30)$$

Here, the division by □ is justified because □ ≠ 0 (modes are unstable).

Now, let the modes be unstable (□ᵣ > 0). Then under the conditions of the theorem, the quantity inside the brackets becomes positive definite, implying, thereby that □ᵢ = 0. This shows that the unstable modes which exist under the conditions of the theorem, are non-oscillatory.

Theorem 3 : The growth rate of arbitrary oscillatory unstable modes, if exist when D´ > 0 lies inside the circle with centre at origin in (□ᵣ – □ᵢ) plane and radius $\sqrt{\frac{D'}{B'}}$.

Proof: Let the oscillatory modes $(\sigma_i \neq 0, \sigma_r > 0)$ exist under the condition D´ > 0. Then □ᵢ ≠ 0 and □ᵣ > 0. Equation (30) yields

$$2\sigma_r A' + B' - \frac{D'}{|\sigma|^2} = 0. \qquad (31)$$

Since A´ > 0, B´ > 0, D´ > 0 and □ᵣ > 0, therefore for the consistency of equation (31), we must necessarily have

$$B' - \frac{D'}{|\sigma|^2} = 0 \qquad \text{or} \qquad |\sigma|^2 < \frac{D'}{B'}.$$

Hence the growth rate of arbitrary unstable oscillatory modes, if exist under the conditions of the theorem lie in a circle with centre at origin and radius $\sqrt{\frac{D'}{B'}}$.

A PARTICULAR CASE : UNIFORM MAGNETIC FIELD

Equation (18), in case of uniform magnetic field, becomes

$$n^2[D(\rho Dw) - k^2 \rho w] + gk^2 (D\rho) w + \frac{\mu_e k_x^2}{4\pi} H_0^2 (D^2 - k^2) w = 0. \qquad (32)$$

Multiplying equation (32) by the complex conjugate w* of w and integrating between z = 0 and z = d, we have

$$n^2 \int_0^d \rho[|Dw|^2 + k^2 |w|^2] dz - gk^2 \int_0^d (D\rho)|w|^2 dz + \frac{\mu_e k_x^2}{4\pi} \int_0^d \left[|Dw|^2 + k^2 |w|^2 \right] dz = 0 \qquad (33)$$

or $$n^2 \int_0^d \rho A dz = gk^2 \int_0^d (D\rho) B dz - \frac{\mu_e k_x^2}{4\pi} \int_0^d \rho A . dz = 0 \qquad (34)$$

where $A = [|Dw|^2 + k^2 |w|^2]$ and $B = |w|^2$.

Theorem 4 : System is stable if the density decreases everywhere in the vertically upward direction, i.e., if (D□) < 0 (stable stratification of density).

Proof : Let (D□) < 0. Then equation (34) does not allow any real value of n. In fact $n^2 < 0$ for (D□) < 0 which ensures that the modes are neutral and oscillatory. Such neutral modes are also termed as neutrally stable because $n^2 > 0$ means that n is real and both positive and negative roots will exist simultaneously, i.e., the existence of a stable mode implies the existence of an unstable mode and vice versa, so the system is always stable for all disturbances.

Theorem 5 : If (D□) > 0 (unstable stratification), system is stable under the condition

$$g(D\rho) < \frac{\mu_e k_x^2}{4\pi} H_0^2.$$

Proof : We rewrite equation (34) as

$$n^2 \int_0^d \rho[|Dw|^2 + k^2 |w|^2] dz = \int_0^d k^2 \left[(gD\rho) - \frac{\mu_e k_x^2}{4\pi} H_0^2 \right] |w|^2 dz - \frac{\mu_e k_x^2}{4\pi} H_0^2 \int_0^d |Dw|^2 dz \qquad (35)$$

and then the same discussion as in Theorem 4 follows when (D□) > 0 and

$$\left[(gD\rho) - \frac{\mu_e k_x^2 H_0^2}{4\pi} \right] < 0.$$

Theorem 6 : If (D□) > 0 (unstable stratification), the system is stable or unstable according as

$$\int_0^d gk^2 (D\rho). B dz < \text{ or } > \frac{\mu_e k_x^2 H_0^2}{4\pi} \int_0^d A . dz$$

Proof : Let (D□) > 0. Then it is clear from equation (34) that if R.H.S. is negative, equation (34) does not allow any positive root of n, so the system is stable under the condition that

$$\int_0^d gk^2 (D\rho). B dz < \frac{\mu_e k_x^2 H_0^2}{4\pi} \int_0^d A . dz$$

On the other hand, if $\int_0^d gk^2 (D\rho). B dz > \frac{\mu_e k_x^2 H_0^2}{4\pi} \int_0^d A . dz$, i.e., the R.H.S. is positive there exists at least one positive value of n. Therefore, system is unstable under the condition

$$\int_0^d gk^2 (D\rho). B dz > \frac{\mu_e k_x^2 H_0^2}{4\pi} \int_0^d A . dz$$

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DUAL CHARACTER OF MAGNETIC FIELD

Consider the exponentially density distribution given by $\rho = \rho_0 \exp(\beta z), \beta > 0.$

Also consider the eigen-function w given by $w = w_0 \sin$ □z.

Then equation (34) shows that the system is stable or unstable according as

$$\frac{gk^2\beta.2\pi^2(e^{\beta d}-1)}{\beta(\beta^2+4\pi^2)(\pi^2+k^2).d} > \text{or} < V_A^2 k_x^2,$$

where $V_A^2 = \dfrac{\mu_e H_0^2}{4\pi\rho} = \dfrac{\mu_e H_1^2}{4\pi\rho_1}$ is the Alfven velocity.

This condition can also be written as

$$\text{or} \quad k_x^2 V_A^2 < \text{or} > L\left(= \frac{gk^2\beta.4\pi^2(e^{\beta d}-1)}{\beta(\beta^2+4\pi^2)(\pi^2+k^2).d}\right)$$

Also, in case of variable magnetic field, the system is stable or unstable according as

$$k_x^2 V_A^2 < \text{or} > M,$$

where $M = \left[\dfrac{4k^2 g\beta}{\beta^2+4\pi^2} + \dfrac{4S^2\pi^2}{d^2}\right].$

A comparative study of the two cases is interesting and is shown below.

For this, consider the difference L – M. Thus,

$$L - M = \frac{4gk^2\beta\pi^2(e^{\beta d}-1)}{\beta(\beta^2+4\pi^2)(\pi^2+k^2)d} - \frac{4k^2 g\beta}{\beta^2+4\pi^2+\dfrac{4S^2\pi^2}{d^2}}$$

$$= \frac{4k^2 g\beta}{\beta d}\left[\frac{\pi^2(e^{\beta d}-1)}{(\beta^2+4\pi^2)(\pi^2+k^2)d} - \frac{\beta d}{\beta^2+4\pi^2+\dfrac{4S^2\pi^2}{d^2}}\right]$$

$$= \frac{4k^2}{d}\left[\frac{\pi^2\left(\beta^2+4\pi^2+\dfrac{4S^2\pi^2}{d^2}\right)(e^{\beta d}-1) - \beta d(\beta^2+4\pi^2)(\pi^2+k^2)}{(\beta^2+4\pi^2)(\pi^2+k^2)\left(\beta^2+4\pi^2+\dfrac{4S^2\pi^2}{d^2}\right)}\right]$$

$$= \frac{4k^2 g}{d}\left[\frac{(\beta^2+4\pi^2)[\pi^2(e^{\beta d}-1) - \beta d(\pi^2+k^2)] + \dfrac{4S^2\pi^2}{d^2}(e^{\beta d}-1)}{(\beta^2+4\pi^2)(\pi^2+k^2)\left(\beta^2+4\pi^2+\dfrac{4S^2\pi^2}{d^2}\right)}\right] \qquad (35)$$

Clearly, L – M > 0 when $k^2 < \frac{\pi^2(e^{\beta d} - 1 - \beta d)}{\beta d}$

L > M implies that a variable magnetic field stabilizes a range which is stable in the presence of a uniform magnetic field. In case

$$k^2 > \left\{\pi^2(\beta^2 + 4\pi^2) + \frac{4S^2\pi^2}{d^2}\right\}\frac{(e^{\beta d} - 1)}{\beta d} - \pi^2$$

or

$$k^2 > \left\{\frac{\pi^2(\beta^2 + 4\pi^2) + \frac{4S^2\pi^2}{d^2}}{\beta d}\right\}(e^{\beta d} - 1) - \pi^2$$

Then L – M < 0 and there exists an unstable range in the presence of a variable magnetic field which is stable in the presence of a uniform magnetic field. This establishes a dual character of variable magnetic field.

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